A Currency Premium Puzzle

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Motivation

- The classical challenge in consumption-based asset pricing is to reconcile a high equity premium with low and smooth risk-free rates.
- Canonical long-run risk and habit models address this tension by inducing a strong, negative correlation between the variance and the mean of the SDF.
  - Whatever increases the volatility of the SDF also decreases its mean.
- This trick has proven highly successful in accounting for closed-economy asset pricing puzzles.

This paper:

- This same trick is also the fundamental reason why these models struggle to account for long-lasting differences in interest rates and currency returns.
- Large currency premia pose a fundamentally different challenge to these models than the classical asset pricing puzzles.

This Paper

- If market are complete:
  - the mean of the SDF governs the expected rate of depreciation.
  - the variance of the SDF governs the return on the currency.
- Because exchange rates are largely unpredictable, all countries must roughly have the same mean SDF.
- Standard habit and long-run risk models then either
  - require either all countries to have the same variance of the SDF, so that there are no differences in expected currency returns or...
  - require that the vast majority of currency returns come from predictable appreciations, which is at odds with the data.
- This tension is near universal in the literature.
- We dub it "the **currency premium puzzle**."

A Highly Successful Trick

Equity premium puzzle: the equity premium is high
Risk-free rate puzzle: the risk-free rate is low and stable.

- Fundamental equation of AP:

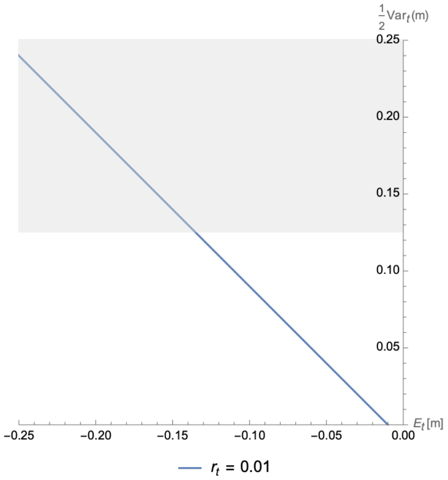
$$1 = \mathbb{E}_t(M_{t+1}R_{t+1})$$

- Risk-free rate (lognormality)

$$r_{f,t} = -\mathbb{E}_t(m_{t+1}) - \frac{1}{2}\,\text{var}_t(m_{t+1})$$

⇒ Pick $\text{var}_t(m_{t+1})$ to match equity premium and $E_t[m_{t+1}]$ for risk-free rate.

- Whatever increases the volatility of the SDF also decreases its mean.
- Stay close to an "iso interest rate line."



Exchange Rates and the SDF

Under complete markets:

- Exchange rate changes (F per H)

$$E_t(\Delta ex_{t+1}) = E_t(m_{t+1}) - E_t(m^\star_{t+1})$$

$\rightarrow$ $\mathbb{E}_t(m_{t+1})$ is directly linked to **expected change in exchange rates!**

- Risk-free rate difference

$$\begin{aligned} r^\star_t - r_t &= \mathbb{E}_t(m_{t+1}) - \mathbb{E}_t(m^\star_{t+1}) \\ &\quad - \frac{1}{2}(\mathsf{var}_t(m^\star_{t+1}) - \mathsf{var}_t(m_{t+1})) \end{aligned}$$

- Currency return:

$$\begin{aligned} \mathbb{E}_t(rx_{t+1}) &= r^\star_t - r_t - \mathbb{E}_t(\Delta ex_{t+1}) \\ &= -\frac{1}{2}(\mathsf{var}_t(m^\star_{t+1}) - \mathsf{var}_t(m_{t+1})) \end{aligned}$$

Two Countries

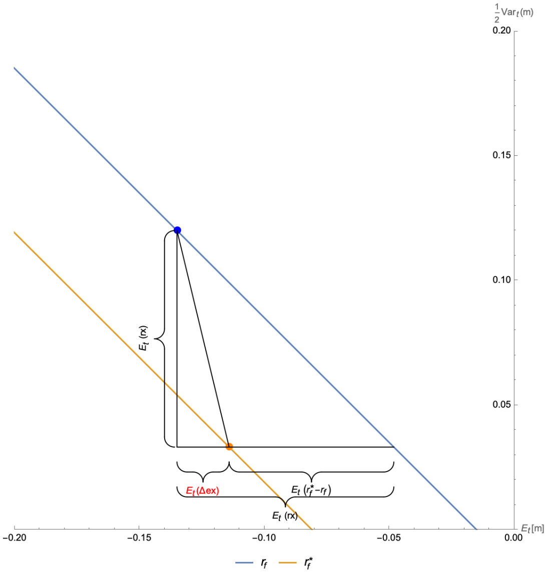
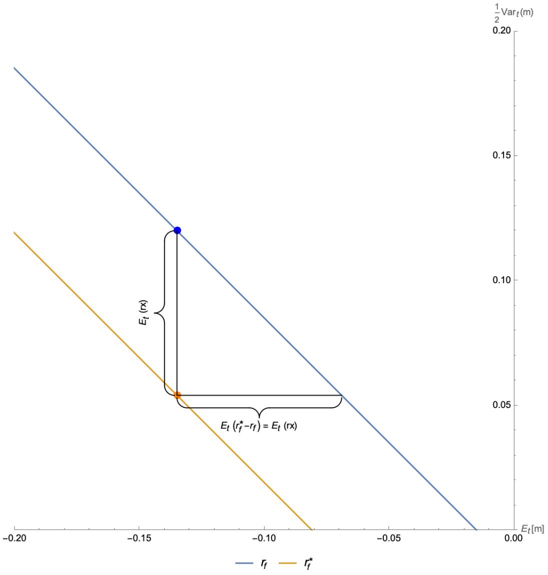
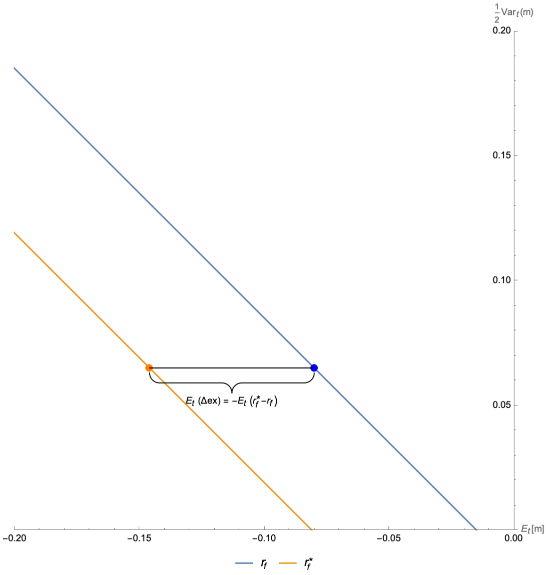


Figure: High r_f currency (red dot) expected to appreciate

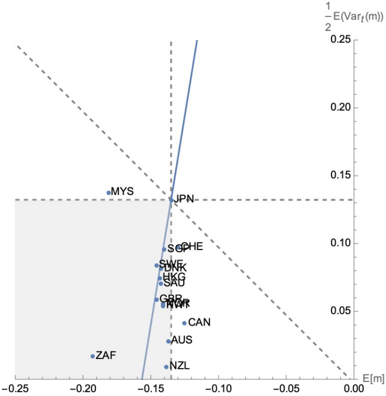
Two Countries: Exchange Rates Unpredictable



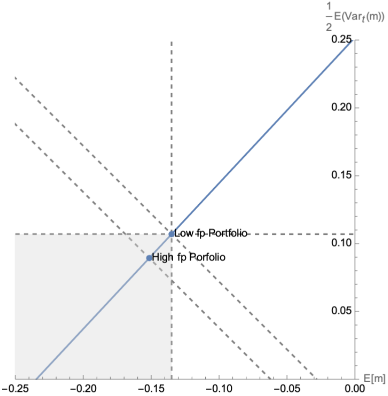
Two Countries: UIP Holds



Data: Currency Returns



Countries



Portfolios of 7 countries

- Exchange rates are largely unpredictable unconditionally; if anything, high interest rate currencies tend to depreciate;
- Portfolios of high interest rate currencies tend to depreciate relative to portfolios with low interest rate currencies.

Implications

- To match the data on currency returns we need:

1. Variance of log SDF much high interest rate country.
2. Means of log SDFs about the same (or, if anything, higher in the higher in the high interest-rate country).

- Canonical models want the high interest rate country to appreciate in expectation.

- Long-run risk models display the currency premium puzzle for unconditional moments.
- External habit models display the currency premium puzzle for conditional moments.

Long-Run Risk

- Epstein-Zin preferences (analogous setup for other country ($\star$))

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

- Consumption growth governed by

$$\begin{aligned} \Delta c_{t+1} &= \mu_i + z_t \\ z_t &= \rho z_{t-1} + \sigma_{LR} \varepsilon_{LR,t} \end{aligned}$$

- Log SDF is given by

$$\begin{aligned} m_{t+1} = \log(\delta) &- \frac{1}{\psi} \Delta c_{t+1} \\ &+ \left(\frac{1}{\psi} - \gamma \right) \left(u_{t+1} - \frac{1}{1-\gamma} \log \left(\mathbb{E}_t [\exp((1-\gamma) u_{t+1}]) \right) \right) \end{aligned}$$

Long-Run Risk: Special Case, International (1/3)

- SDF unconditionally

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\mathsf{var}_t(u_{t+1}))$$

$$\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\mathsf{var}_t(u_{t+1}))$$

- Unconditional long-run risk line

$$\frac{1}{2}\,\mathbb{E}(\mathsf{var}_t(m_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\,\mathbb{E}(m_{t+1}) + const$$

- All countries have SDF on blue line.

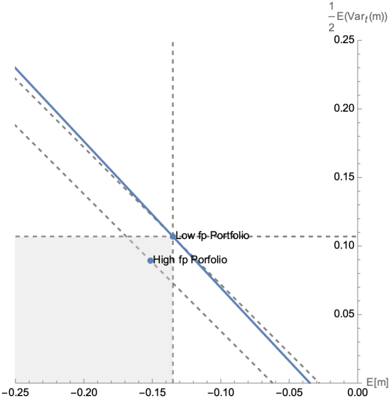


Figure: Long-run risk line

Long-Run Risk: Special Case, International (2/3)

- Unconditional expectation of risk-free rate difference

$$\mathbb{E}(r^{\star}_{f,t} - r_{f,t}) = -\frac{1}{2}\left(\frac{1}{\psi} - 1\right)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}\left(\mathsf{var}_t(u_{j,t+1}) - \mathsf{var}_t(u_{i,t+1})\right)$$

- Unconditional expectation of change in exchange rate

$$\mathbb{E}(\Delta ex_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\,\mathbb{E}\left(\mathsf{var}_t(u^{\star}_{t+1}) - \mathsf{var}_t(u_{t+1})\right)$$

- And the unconditional expectation of currency return is given by

$$\begin{aligned}\mathbb{E}(rx_{t+1}) &= \mathbb{E}(r^{\star}_{f,t} - r_{f,t}) - \mathbb{E}_t(\Delta ex_{t+1}) \\ &= -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}\left(\mathsf{var}_t(u^{\star}_{t+1}) - \mathsf{var}_t(u_{t+1})\right)\end{aligned}$$

- We have:

$$\mathbb{E}(\Delta ex_{t+1}) = -\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}\,\mathbb{E}(rx_{t+1}) = -\frac{\gamma - 1}{1 - \frac{1}{\psi}}\,\mathbb{E}(r^{\star}_{f,t} - r_{f,t})$$

- Chen, Favilukis and Ludvigson (2013) and Bansal, Kiku and Yaron (2016) estimate $\psi > 1$ and $\gamma >> 1/\psi$.
- Expected changes in exchange rates exceed interest rate differentials.

Long-Run Risk: Special Case, International (3/3)

- Assume: $\gamma > 1$, as well as ϕ, μ, δ and ρ are symmetric across countries. No short-run risk in the sense that $\sigma_{SR,t} = 0$ for all t.

Proposition

If agents prefer early resolution of uncertainty so that $\gamma > 1/\psi$, then

- If $\psi < 1$, $\mathbb{E}(rx_{t+1}) = \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}} \, \mathbb{E}(r^{\star}_{f,t} - r_{f,t})$ has opposite sign of $\mathbb{E}(r^{\star}_{f,t} - r_{f,t})$: high interest rate currency yields negative currency premium.
- If $\psi = 1$, $\mathbb{E}(r^{\star}_{f,t} - r_{f,t}) = 0$, interest rates are identical across countries;
- If $\psi > 1$, $-\mathbb{E}(\Delta ex_{t+1}) = \frac{\gamma - 1}{1 - \frac{1}{\psi}} \, \mathbb{E}(r^{\star}_{f,t} - r_{f,t})$, high interest rate currency appreciates unconditionally. Furthermore, if $\gamma > 2 - \frac{1}{\psi}$, $\frac{-\mathbb{E}(\Delta ex_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{\gamma - 1}{\gamma - \frac{1}{\psi}} > \frac{1}{2}$: appreciation of the high interest currency accounts for more than 50% of the currency premium.

Long-Run Risk: The CCGR Model

- Model taken from Colacito, Croce, Gavazzoni, and Ready (2018).
- Endowments in country i:

$$y_{i,t} = \mu + y_{i,t-1} + z_{i,t-1} - \tau(y_{i,t-1} - \frac{1}{N}\sum_j y_{j,t-1}) + \varepsilon_{i,t}^{SR}$$

- Long-run risk

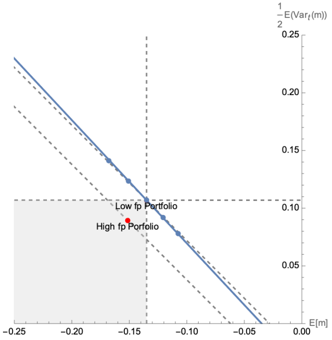
$$z_{i,t} = \rho z_{i,t-1} + (1 + \beta_i^z)\varepsilon_{global,t}^{LR} + \varepsilon_{i,t}^{LR}$$

- Consumption bundle

$$C_t^i = (I_{i,t}^i)^\alpha \left(I_{j,t}^i\right)^{1-\alpha}$$

- CCGR considers five countries with β_i^z ranging from -0.65 to 0.65.

Long-Run Risk: The CCGR Model in the SDF Graph



- Unconditionally, all countries lie on the blue line.
- Currency premia (vertical difference) are tightly linked to expected change in exchange rates (horizontal difference).

The CCGR Model: Closed-form solutions

- Solve a two-country CCGR model with risk-adjusted affine approximation.

$$\mathbb{E}(r_t^\star - r_t) = \theta\left((\beta^z)^2 - (\beta^{\star z})^2\right)\left(1 - \frac{1}{\psi}\right)\left(\sigma_{global}^{LR}\right)^2$$

$$\mathbb{E}(\Delta ex_{t+1}) = -\theta\left((\beta^z)^2 - (\beta^{\star z})^2\right)(\gamma - 1)\left(\sigma_{global}^{LR}\right)^2$$

$$\mathbb{E}(rx_t) = \theta\left((\beta^z)^2 - (\beta^{\star z})^2\right)\left(\gamma - \frac{1}{\psi}\right)\left(\sigma_{global}^{LR}\right)^2$$

  where $\theta > 0$ is a constant.
- Obviously

$$\mathbb{E}(\Delta ex_{t+1}) = -\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}\,\mathbb{E}(rx_{t+1}) = -\frac{\gamma - 1}{1 - \frac{1}{\psi}}\,\mathbb{E}(r_{f,t}^\star - r_{f,t})$$

- Under CCGR calibration ($\gamma = 6.5$, $\psi = 1.6$), appreciation of high interest rate currency accounts for 93.62% of currency premium unconditionally.

Comparison of Asset Pricing Moments

	Data	BY	BS-R	BS-N	CC2013	CCGR
			Panel A: Domestic			
slope(U)	2.12	-1.04	-1.04	-1.04	-1.05	-1.07
			Panel B: International			
Carry Trade	4.96	-	0.25	0.05	0.06	4.51
fp	7.11	-	0.23	0.43	0.29	1.65
$-\Delta ex$	-2.15	-	0.02	-0.38	-0.23	2.86
Sharpe Ratio	0.54	-	0.01	0.00	0.00	0.05
β^{ct}	$0.68^{\star\star}$	-	1.02	0.09	0.72	$4.64^{\star\star}$
	(0.27)	-	(6.52)	(4.15)	(2.73)	(1.95)
β^{fama}	$1.81^{\star\star\star}$	-	2.21	-0.51	2.04	$6.91^{\star\star\star}$
	(0.53)	-	(9.49)	(7.23)	(5.12)	(2.59)

Notes: BY: Bansal and Yaron (2004); BS-R: Bansal and Shaliastovich (2013), real; BS-N: Bansal and Shaliastovich (2013), nominal, CC2013: Colacito and Croce (2013)

External Habit: Setup (1/2)

- Habit utility (analogous equations for country $(^\star)$)

$$\mathbb{E}\sum_{t=0}^{\infty}\delta^t\frac{(C_t-H_t)^{1-\gamma}-1}{1-\gamma}$$

- Following Campbell and Cochrane (1999), we define the surplus consumption ratio as

$$X_t \equiv \frac{C_t-H_t}{C_t}$$

- The pricing kernel is given by

$$M_{t+1}=\delta\left(\frac{X_{t+1}}{X_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

- Consumption growth follows

$$\Delta c_{t+1}=\mu+\sigma\varepsilon_{t+1}$$

- Shocks can be correlated across countries.

External Habit: Setup (2/2)

- Assume a log surplus consumption ratio of

$$x_{t+1} = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - \mu)$$

- with a sensitivity function $\lambda(x_t)$

$$\lambda(x_t) = \begin{cases} \frac{1}{\bar{X}}\sqrt{1-2(x_t-\bar{x})} - 1 & \text{when } x < x_{max} \\ 0 & \text{elsewhere} \end{cases}$$

- where surplus consumption ratio has steady-state $\bar{X}$

$$\bar{X} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$$

- and its log an upper bound of x_{max}

$$x_{max} = \bar{x} + \frac{1-\left(\bar{X}\right)^2}{2}$$

- Note that $\gamma(1-\phi) - B > 0$ for existence of steady state.
- Parameter B nests different SDFs from the literature.

External Habit with constant risk-free rate

- If $B = 0$ (as in Campbell and Cochrane (1999)), risk-free rate is constant

$$r_{f,t} = -\log(\delta) + \gamma\mu - \frac{1}{2}(\gamma(1-\phi))$$

- External habit SDF is a dot on iso-riskfree rate line.

$$\frac{1}{2}\text{var}_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) + const$$

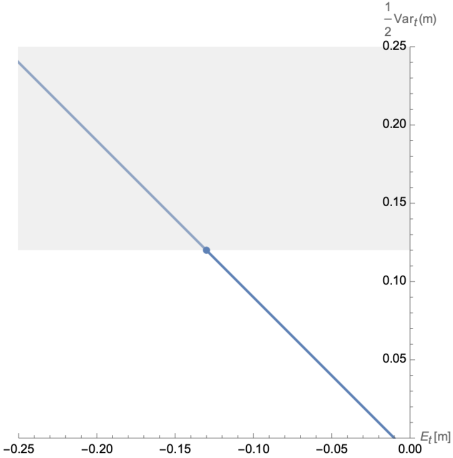


Figure: Illustration of the case with $B = 0$.

External Habit with time-varying risk-free rate

- If $B > 0$ (as in Wachter (2006)),

$$r_{f,t} = -\log(\delta) + \gamma\mu - \frac{1}{2}(\gamma(1-\phi) - B) \\ - B(x_t - \bar{x})$$

- The set of SDFs lie on

$$\frac{1}{2}\text{var}_t(m_{t+1}) = -\frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}\mathbb{E}_t(m_{t+1}) \\ + const$$

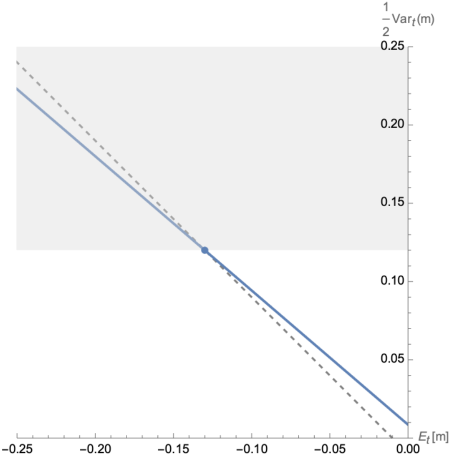


Figure: Wachter (2006)

External Habit: Verdelhan (2010)

- If $B < 0$ (as in Verdelhan (2010)

$$r_{f,t} = -\log(\delta) + \gamma\mu - \frac{1}{2}(\gamma(1-\phi) - B) - B(x_t - \bar{x})$$

- The set of SDFs lie on

$$\frac{1}{2}\mathsf{var}_t(m_{t+1}) = -\frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}\mathbb{E}_t(m_{t+1}) + const$$

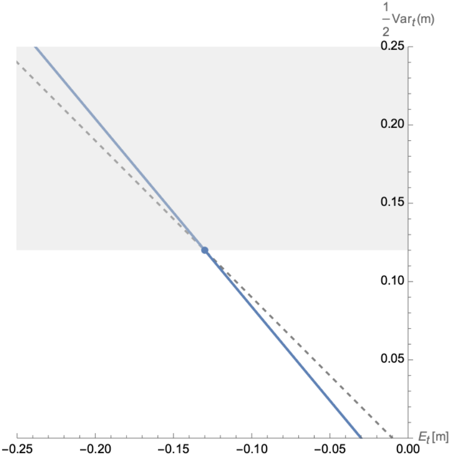


Figure: Verdelhan (2010)

External Habit: International

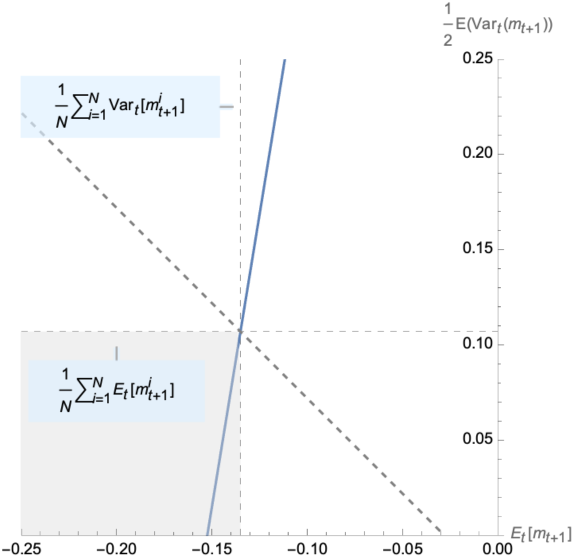
Proposition

In an external habit model with two symmetric countries,

- If $B > 0$, $\mathbb{E}_t(rx_{t+1}) = -\frac{\gamma(1-\phi)-B}{B}\,\mathbb{E}_t(r^{\star}_{f,t} - r_{f,t})$ is of the opposite sign as $\mathbb{E}_t(r^{\star}_{f,t} - r_{f,t})$: high interest rate currency yields negative currency premium conditionally.
- If $B = 0$, $r^{\star}_{f,t} - r_{f,t} = 0$, the model cannot generate differences in risk-free rates.
- If $B < 0$, $-\mathbb{E}_t(\Delta ex_{t+1}) = \frac{\gamma(1-\phi)}{B}(r^{\star}_{f,t} - r_{f,t})$, high interest rate currency appreciates conditionally. Furthermore, if $\gamma(1-\phi) > -B$, $\frac{-\mathbb{E}(\Delta ex_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B} > \frac{1}{2}$: Appreciation of the high interest currency accounts for more than 50% of the currency premium.

Data: Conditional

- Data displays similar patterns conditionally as uncondionally.



Comparison of Asset Pricing Moments

	Data	CaCo	Verdelhan	Stathopolous
		Panel A: Domestic		
slope(C)	2.12	-1.00	-2.00	-
		Panel B: International		
Carry Trade	4.96	-	4.47	-1.34
fp	7.11	-	2.24	1.18
$-\Delta ex$	-2.15	-	2.22	-2.52
Sharpe Ratio	0.54	-	0.06	-0.06
β^{ct}	$0.68^{\star\star}$	-	$2.14^{\star\star\star}$	$-1.87^{\star\star}$
	(0.27)	-	(0.16)	(0.81)
β^{fama}	$1.81^{\star\star\star}$	-	$2.18^{\star\star\star}$	$-2.20^{\star}$
	(0.53)	-	(0.23)	(1.20)

Notes: CaCo: Campbell and Cochrane (1999); Verdelhan: Verdelhan (2010), Stathopoulos: Stathopoulos (2017)

Conclusion

- Canonical models with long-run risk and external habits models link the first and second moments of the SDF.
- This feature helps to resolve domestic equity premium and risk-free rate puzzles.
- But it leads to a currency premium puzzle in an international context.
- Internationally, these models imply high interest rate currencies to either generate negative currency return or predictably appreciate.
- Both of these predictions are at odds with the data.